From prl@aps.org Fri Apr 30 14:33:18 2004
Date: Fri, 30 Apr 2004 12:26:15 UT
From: Physical Review Letters <prl@aps.org>
To: rbb11@phy.cam.ac.uk
Subject: Your_manuscript LP9475 Blumenfeld

Re: LP9475
Stresses in isostatic granular systems and emergence of force chains
by Raphael Blumenfeld

Dr. Raphael Blumenfeld
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Cambridge University
Madingley Road
Cambridge CB3 0HE
UNITED KINGDOM

Dear Dr. Blumenfeld,

The above manuscript has been reviewed by our referees. We ask you to consider the enclosed comments from the reports.

While we cannot make a definite commitment, the probable course of action if you choose to resubmit is indicated below.

( ) Acceptance, if the editors can judge that all or most of the criticism has been met.

(X) Return to referee B for review if available.

( ) Submittal to new referee(s) for review.

Please accompany any resubmittal by a summary of the changes made, and a brief response to all recommendations and criticisms.

Yours sincerely,

Reinhardt B. Schuhmann
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Report of Referee A -- LP9475/Blumenfeld

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The manuscript describes a model of forces within two dimensional granular materials.

The granular systems are isostatic and the model relies on an equation that gives a connection between the stress tensor and the (microscopic) fabric tensor. Initially results show that in this regime the stress equations are hyperbolic (with a possible exception at granular scales). A solution of the stress equations, with boundary load conditions, reveals features that have been identified as force chains. As part of a discussion relating this special case to general granular packings the author identifies the isostatic state as a critical point at which the length scale for granular rearrangements diverges. The author proposes that away from this critical point granular systems become two phase systems with elastic domains within an isostatic matrix.

The account is reasonably well presented and readable although terse. It has some elements of generality that may appeal to non specialist readers and concerns a subject of current debate amongst practitioners in granular materials. Some aspects of the account could be improved by further material or explanations (a) How does the fabric tensor introduced in eq (2) relate to traditional forms involving normal



contact vectors? (I would expect the trace of the average tensor to be related to packing and coordination?). (b) Can the loops presented in figure 1 be given a physical interpretation? If not can their numbers and statistics be related to more natural microstructural descriptors? (c) At several points e.g. p3 "which appear to be only a few grains wide" the account discusses the size of 'chains'. With respect to emergence what constitutes a chain (since connectivity is ensured)? (d) I can not reproduce eq. (4)? In fact it appears to be dimensionally inconsistent? A few more details would makea major improvement. (e) In what way does eq(6) give evidence against the parabolic form that eq(4) does not? (f) The factor bxy on p 6 (top) and the scale d in equation 12 appear without sufficient explanations? (g) It is insufficient to say that a critical point "has already been entertained in the literature" P 9 (top). How does this add to the argument?

I recommend publication in Physical Review Letters after these points have been addressed.

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Report of Referee B -- LP9475/Blumenfeld

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This paper deals with stress transmission in granular packings. In a first part, the author derives closed equations for the stress components. An important question about equations governing stress transmission is whether they are elliptic (as in elasticity theory), parabolic or hyperbolic. The author claims to show that the equations are hyperbolic on macroscopic scales. Discussing the solution of the equations in some simple cases, the author explains the appearance of force chains from the simple form of the equations obtained. Finally the author proposes an experimental test for this theory, based on the measurement of the local fabric tensor. The paper is rather well written and easy to follow. However, it is based on the fundamental equation (6), which derivation is not clear in the paper. Indeed, Eq. (5) is a consequence of the change of variables (5) in Eq. (4). However, this change of variables seems to be valid only when the fabric tensor is uniform. The author should clarify this point: is Eq. (6) valid even when the fabric tensor is not uniform (in that case, the author should explain a bit how the algebra leads to this remarquable result)? or is it valid only when the fabric tensor is uniform? This question of the uniformity of the fabric tensor is of first importance, as granular packings are usually studied in a disordered state (or at least weekly disordered), in which case this tensor is not uniform. Due to this uncertainty at the starting point of the paper, I cannot recommend its publication in Physical Review Letter in its present form. However, as Eq. (5) would be an important result if true in general in granular packing I suggest to the author to rewrite the derivation of Eq. (5) in a more convincing manner, in which case the possibility of publication of this paper could be considered.

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Report of Referee C -- LP9475/Blumenfeld

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The specification of both stress and stress gradient as boundary conditions is unusual but, apparently, necessary for isostatic systems. A comment might be included regarding this.